Vol. XI & Issue No. 7 July - 2018
INDUSTRIAL ENGINEERING JOURNAL

COMPARISON OF PROJECT SCHEDULING TECHNIQUES: PERT VERSUS MONTE CARLO SIMULATION

Pooja Deshmukh
N. R. Rajhans

Abstract:

With an increase in the focus on achieving customer satisfaction, manufacturing industries are aiming to optimise their processes to a great extent. In any project the constraints of schedule, budget, scope and quality which form the basis of the project management triangle can be fulfilled by implementing project management planning tools & techniques appropriately. In this research study PERT (Program Evaluation and Review Technique) is applied on a project to evaluate the probability of project completion. Another scheduling tool which has gained popularity in recent times is the Monte Carlo simulation. This technique is applied on the same project to perform schedule risk analysis by evaluating the criticality index. The results of both the techniques are compared using hypothesis test to evaluate the more suitable one which can be used practically as a scheduling tool.

***KEYWORDS:** Project Management triangle, PERT, Monte Carlo simulation, Hypothesis test*

1. INTRODUCTION

In order to complete a project within schedule it is crucial to estimate the probability of project completion precisely. One of the factors that can affect this estimation is activity schedule uncertainties. Thus it is necessary to consider these uncertainties while evaluating the project completion time. The most commonly used tools for scheduling a project are CPM & PERT. Liu Jun – Yan (2012) reviewed the available techniques of schedule uncertainty (CPM, PERT, Monte Carlo simulation) and analysed the advantages & disadvantages of existing research in this field. K. R. Mac Crimmon & C. A. Ryavec (1963) analysed the aspects of the PERT model mathematically. They obtained an indication of the magnitude of errors introduced by the assumptions in the model & suggested possible modification & improvements. M. A. A. Cox (1995) proposed a method to obtain the project completion time by assuming the duration of activities follow a normal distribution.

To complete a project within a predefined schedule, it is essential to use proper planning tools and techniques. The research study proposes a comparison between two most widely used project planning & scheduling techniques PERT (Program Evaluation and Review Technique) & Monte Carlo simulation. PERT considers the uncertainties in activity durations by considering 3 estimates of time. In spite of this the results obtained from PERT have a deviation from practical project completion time. On the other hand, in Monte Carlo simulation a distribution for activity duration can be selected and a range of probability of project completion can be obtained based on number of simulation runs. Hypothesis test is carried out to evaluate the results obtained by both the techniques and select the more suitable one. The study shows that the planning & scheduling techniques assist project managers to estimate the probability of project completion within a schedule efficiently.

2. LITERATURE REVIEW

A project consists of activities to be executed in a predefined sequence in order to complete the project within schedule. There are certain project management tools & techniques that assist project managers to schedule the project in a precise manner. Wyrozębski P & Wyrozębska A. (2013) compared probabilistic techniques of project planning & scheduling – PERT, GERT and Monte Carlo simulation. They found that an integrated approach using Monte Carlo simulation along with PERT results in a higher reliability of schedule planning. W. Na, P. Wuliang & G. Hua (2014) evaluated the project plan robustness and presented a complete estimation of project plan using Monte Carlo simulation. They suggested that the approach could assist project managers to determine the project duration risk & identify key tasks that influence the project plan robustness at the start of project planning. Z. Kong, J. Zhang, Chao Li, X. Zheng and Q. Guan (2015) suggested that Monte Carlo simulation can provide direct pictorial information which could assist the decision makers to select a realistic project completion time. B. McCabe (2003) developed a probabilistic model to estimate lower and upper duration estimates required in the preparation of a schedule risk analysis using Monte Carlo simulation and discussed the lessons learned. A. Connor & S. Mac Donell (2006) described a model to link estimates of project duration to a historical database of a software project by using Monte Carlo simulation. H Arsham (1993) presented a non-statistical approach (What-if analysis) to analyze various types of activity duration uncertainties in a project. J. K. Visser (2016) investigated the output of schedule simulations when different distributions were used to express the uncertainty in activity duration. He applied the hypothesis test (t-test) and found that there is no significant difference in the output distributions when different input distributions with the same mean and variance values are used. M. Hajdu & O. Bokor (2014) applied various distributions to projects & investigated their effects on project duration. They found that the usage of different activity distributions did not result in significant differences from a practical point of view. The precision of the 3 point in determining the distribution of the project duration estimation plays a more important role. M. Hajdu & O. Bokor (2016) applied Monte Carlo simulation to analyze effect of various probability distributions for activity duration. The analysis showed that + or – 10% difference in the PERT 3 point estimation causes greater deviation in the calculated probability of project completion than the type of activity duration distribution. Thus project managers should devote more effort to precisely determine the activity durations. According to A. A. Opaleye, O. E. Charles-Owaba & B. Bender (2017) suggested that to solve the problems of project delay, statistical distributions have to be selected combined with historical data of duration of activity.

M. M. Skrtic and K. Horvatincic (2014) performed a comparative study of quantitative risk analysis that have an impact on the cost or time including sensitivity analysis, PERT, Monte Carlo simulation, Decision tree, Brainstorming & Delphi method. Many researchers have studied the drawbacks of PERT to consider uncertainties in activity durations. On the other hand research studies have also been carried out on the use of Monte Carlo simulation to estimate the probability of project completion. K. Doubravský and R. Doskočil (2015) compared PERT & Monte Carlo simulation for the calculation of probability of project completion. The probabilities were compared using statistical hypothesis testing. It was concluded that there is difference between the approaches from application's point of view. M. Karabulut (2017) studied a project execution tracking system. Traditional CPM and PERT methods, and Monte Carlo simulation as risk analysis tool were used for scheduling. The results showed that Monte Carlo simulation gave more realistic outcomes. C RAGSDALE (1989) demonstrated the advantages of Monte-Carlo simulation over the traditional PERT/CPM techniques. S. Tattoni & M. M. Schiraldi (2008) showed through an algorithm and experimental results that the computational time which is historically the major drawback of Monte Carlo simulations, is definitely minimum these days due to the computational power available. W. Tysiak (2011) showed how to overcome the disadvantages of the PERT approach by using Monte Carlo simulation. He found that PERT introduces insecurities in project planning, whereas Monte Carlo simulation is comparatively more precise. K. A. Kirytopoulos, V. N. Leopoulos and V. K. Diamantas (2008) used PERT and Monte Carlo Simulation for project scheduling and the results produced under four different scenarios were compared. They found that results obtained by Monte Carlo Simulation were superior to that of PERT.

3. METHODOLOGY

The current study is carried out for a project process of a manufacturing industry. In order to complete a project within schedule, project management tools and techniques assist project managers to a great extent. One such planning tool is the PERT (Program Evaluation & Review Technique). It assists project managers to estimate the probability of project completion. Figure 1 shows the methodology used in this research study.

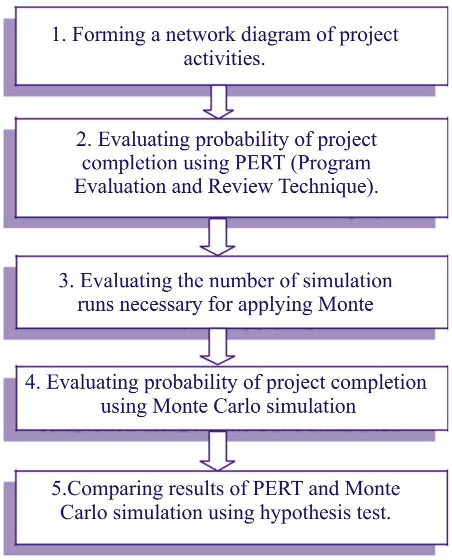


Figure 1: Methodology

A network diagram is formed for a project process of a manufacturing industry. PERT is applied to the project process in order to evaluate the probability of project completion. The results obtained showed that PERT has certain limitations due to which it is difficult to implement it practically. These limitations include the accurate estimation of activity durations. Monte Carlo simulation is then applied on the same process to evaluate the probability of project completion. A comparison of both the techniques is carried out using the hypothesis test (t-test) to verify whether the results obtained by both the methods are similar or not.

4. CASE STUDY

4.1 PERT (Program Evaluation and Review Technique)

It is a statistical tool, which was designed to analyze and represent the tasks involved in completing a given project and used in project management to evaluate the probability of project completion. PERT considers a 3 time estimate of activity durations namely: Pessimistic, most likely and optimistic time. The 3 estimate activity durations along with predecessor activities and activities mean and variance are mentioned in table 1.

Table 1: Activity duration & Predecessor

Activity	Immediate Predecessors	Pessimistic estimate (p)	Most likely estimate (m)	Optimistic estimate (o)	Mean μ	Variance σ2
A	-	15	12	10	12	0.69
B	A	3	1	1	1	0.11
C	B	4	1	1	2	0.25
D	B	7	3	2	4	0.69
E	B	9	4	3	5	1.00
F	C, D, E	12	5	4	6	1.78
G	F	4	1	1	2	0.25
H	G	5	1	1	2	0.44
I	H	4	1	1	2	0.25
J	H	13	7	5	8	1.78
K	H	12	4	3	5	2.25
L	I, J, K	10	4	3	5	1.36
M	L	6	2	1	3	0.69
N	M	7	3	2	4	0.69
O	N	3	2	1	2	0.11
P	O	3	1	1	1	0.11
Q	O	5	2	2	3	0.25
R	P, Q	3	1	1	1	0.11

Activity on arrow network diagram is constructed using the above details to evaluate the critical path as shown in figure 2.

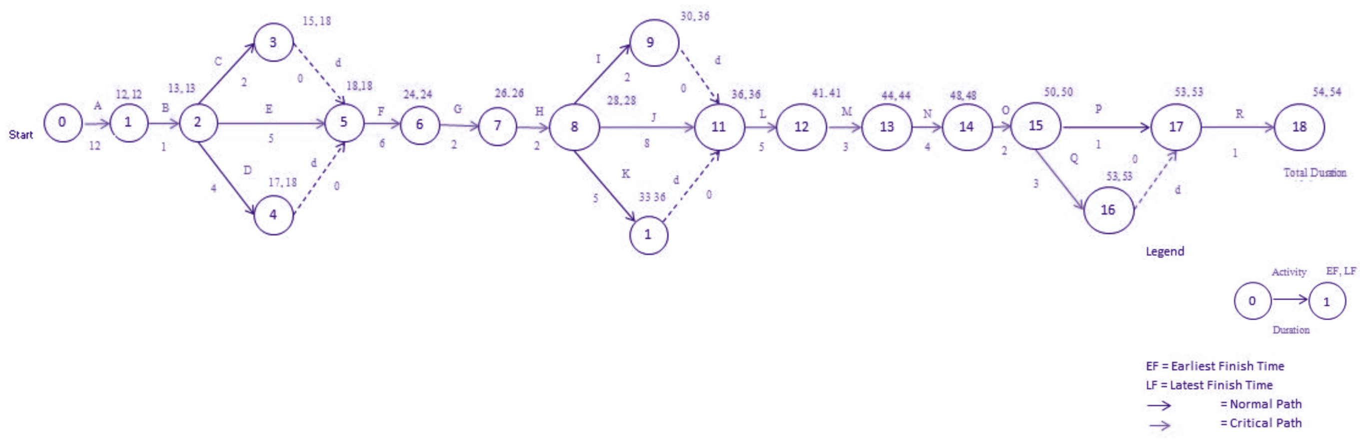


Figure 2: Network Diagram

- The Critical Path is A – B – E – F – G – H – J – L – M – N – O – Q – R and the project completion duration is 54 days.
- Evaluation of probability of Project completion in **60 days**: Let,

T = Project duration (in days), which has (approximately) a normal distribution with a mean μ of 54 and a variance σ^2 of 9

d= deadline for the project = 60 days

Since the standard deviation σ of T is 3, the number of standard deviations by which d exceeds μ is shown in equation 1.

$$K\alpha = \frac{d-\mu}{\sigma} \qquad (1)$$

$$K\alpha = 2$$

Therefore, using Table for a *standard* normal distribution (a normal distribution with mean 0 and variance 1), the probability of meeting the deadline is shown in equation 2

$P(T \leq d) = P$ (standard normal $\leq K$) (2)

= 1 - P (standard normal $\geq K$)

= 1- 0.02275

= 0.9772 or 97.72%

This $P(T \leq d)$ is only a rough approximation of the true probability of meeting the project deadline.

4.2 Monte Carlo Simulation

- In a Monte Carlo simulation, each input is varied within a predefined range hundreds of times to generate a range of outputs along with the frequency of occurrence. This frequency is then translated into the probability of the respective output's occurrence. By using Monte Carlo simulation, we can generate a mathematical distribution (often a bell curve) showing the likely range of outcomes. In this research study an excel spreadsheet simulation is used to calculate the total project critical-path duration and probability of project completion within a predefined schedule.

- Defining distributions for activity time:

  Duration of activities in a project have uncertainties which need to be considered to evaluate the project completion probability. In order to consider these uncertainties in activity duration, a standard normal distribution is defined for every activity.

- Evaluating number of simulation runs needed:

  a. Method 1: This method consists of applying a formula shown in equation 3 to calculate the number of runs as shown below: E. Bukaçi & Th. Korini (2016)

$$n = \left(\frac{100 * Zc * Sx}{\mu * E} \right)^2$$

Where n = number of simulation runs Zc = 1.96 (Value of confidence coefficients)

E = 0.5 (Error of the mean)

= 54 (Mean of the sample)

Sx = 3 (Standard deviation of the sample).

Substituting the values in equation 3 we get

n = 1317

If the simulation is run for 1317 iterations, we are 95% confident that the calculated mean will not differ by more than 0.5% from the true mean.

b. Method 2: Simulation runs are slowly increased from 5, 10 up to a value where there is a small change or no change in the output as shown in Table 2 This value of the run is then considered as the optimum value to obtain desired results.

Table 2: Runs & Average Days

Runs	5	10	20	30	40	50	60	70
Avg Days	49.6	51.5455	51.1905	50.7419	51.1951	51.4314	51.4262	51.6056
Runs	80	90	100	500	1000	1300	1500	5000
Avg Days	51.7	51.824	51.673	51.6627	51.5590	51.6261	51.6646	51.7428

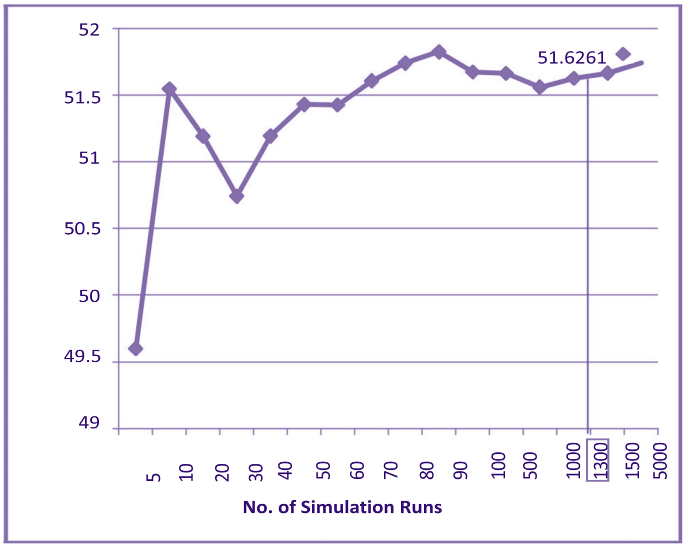


Figure 3: No. of Simulation runs

From the above graph in figure 3 it is seen that when the number of simulation runs = 1320 the values reach a saturation point.

Therefore by both the method 1 & 2 it is seen that the optimum number of simulation runs to get the desired output is approximately 1320 runs.

- Monte Carlo Simulation: A total of 1320 runs are considered for executing Monte Carlo simulation in Microsoft Excel. Table 3 shows the frequency, probability & % cumulative probability of project completion for particular number of days.

Table 3: Frequency, Probability & % Cumulative Probability

Days	Freq	Probability	Cum probability	% Cum probability
40	0	0.0000	0	0
41	1	0.0008	0.0008	0.07575
42	2	0.0015	0.0023	0.22727
43	4	0.0030	0.0053	0.53030
44	1	0.0008	0.0061	0.60606
45	13	0.0098	0.0159	1.59090
46	32	0.0242	0.0402	4.01515
47	41	0.0311	0.0712	7.12121
48	82	0.0621	0.1333	13.3333
49	91	0.0689	0.2023	20.2272
50	151	0.1144	0.3167	31.6666
51	180	0.1364	0.4530	45.3030
52	179	0.1356	0.5886	58.8636
53	164	0.1242	0.7128	71.2878
54	126	0.0955	0.8083	80.8333
55	86	0.0652	0.8734	87.3484
56	87	0.0659	0.9393	93.9393
57	40	0.0303	0.9696	96.9696
58	25	0.0189	0.9886	98.8636
59	11	0.0083	0.9969	99.6969
60	4	0.0030	1	100
61	0	0.0000	1	100
62	0	0.0000	1	100
63	0	0.0000	1	100
64	0	0.0000	1	100

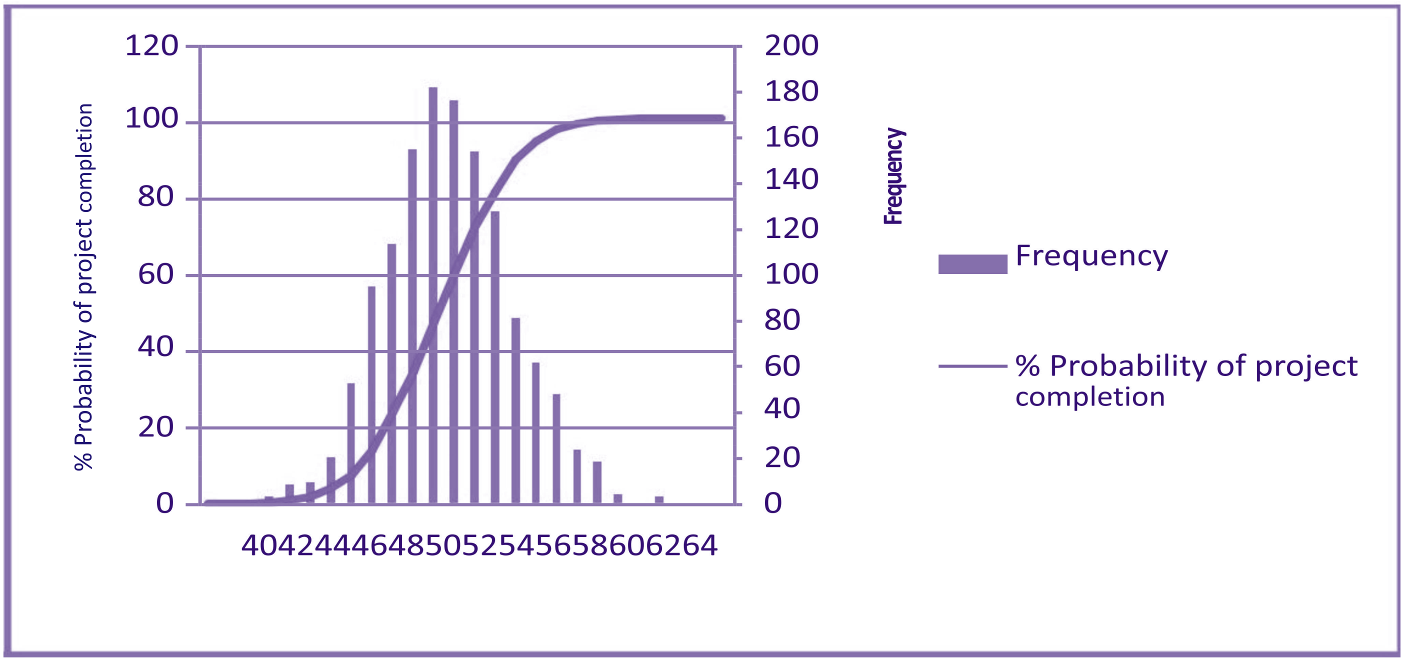


Figure 4: % Probability of Project Completion & frequency of duration in 1320 runs

Figure 4 shows the % probability of project completion and frequency of occurrence of duration in days for 1320 simulation runs.

- Criticality Index: The output of schedule risk analysis is a set of measure that defines the degree of activity criticality. This measure refines the black-and-white view of the critical path to a degree of sensitivity, as follows:

Criticality Index (CI): Measures the probability that an activity is on the critical path as shown in equation 4.

$$\text{Criticality Index} = \frac{\text{No. of runs for which an activity lies on the critical path}}{\text{Total number of simulation runs}} \quad (4)$$

Table 4: Criticality Index

Sr. No.	Activity	No. of runs activity is on critical path	Criticality Index
1	A	1320	1
2	B	1320	1
3	C	0	0
4	D	473	0.3586
5	E	1106	0.8382
6	F	1320	1
7	G	1320	1
8	H	1320	1
9	I	0	0
10	J	1239	0.9386
11	K	140	0.1064
12	L	1320	1
13	M	1320	1
14	N	1320	1
15	O	1320	1
16	P	327	0.2476
17	Q	1320	1
18	R	1320	1

Table 4 shows the critical index obtained for all 18 activities. The total No. of simulation runs is 1320.

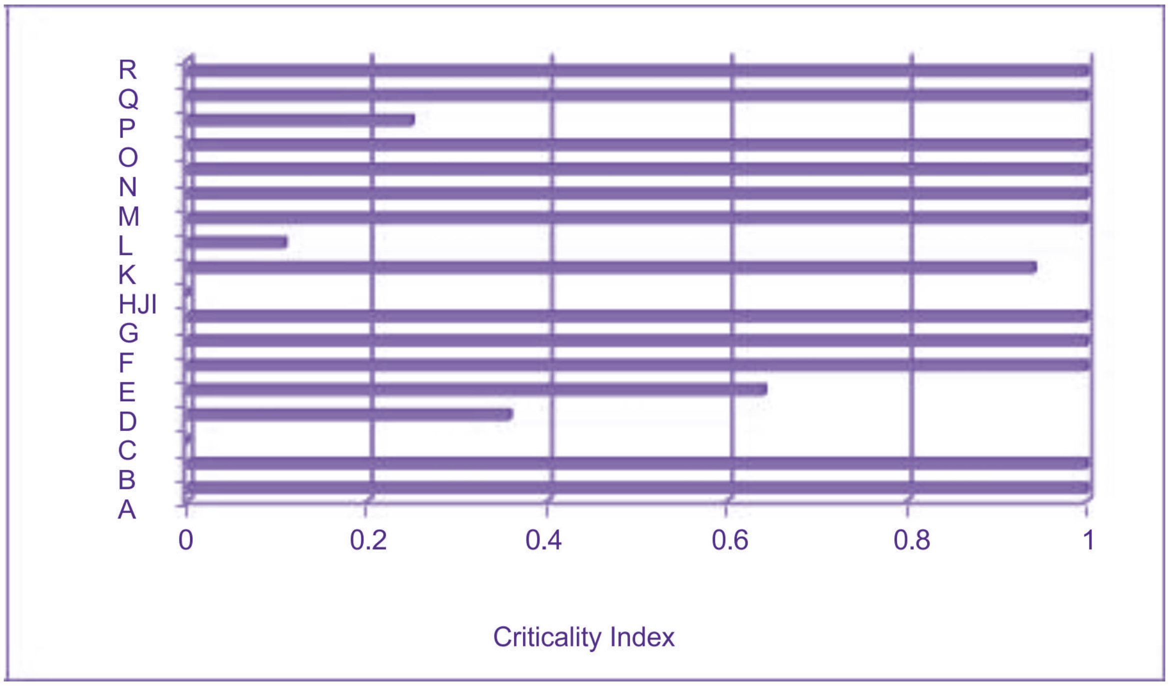


Figure 5: Criticality Index

Figure 5 shows the graphical representation of the criticality index of the project activities. It can be observed that activities A, B, D, F, G, H, L, M, N, O, Q and R have the highest criticality index 1. This signifies that these activities will lie on the critical path irrespective of the number of simulation runs. While the other activities have a criticality values range in between 0 to 1. This signifies that the number of times the activities will lie on the critical path varies as per the number of simulation runs.

5. RESULTS:

The results obtained by both the techniques i.e. PERT & Monte Carlo simulation are compared to evaluate the more suitable one using hypothesis testing. Formulation of hypothesis is shown below:

- **Null Hypothesis**: The means of the two samples are the same i.e. the results obtained from both the methods have an insignificant difference.

- **Alternative Hypothesis**: There is a significant difference in the mean.

Rules for Rejecting the Null Hypothesis:

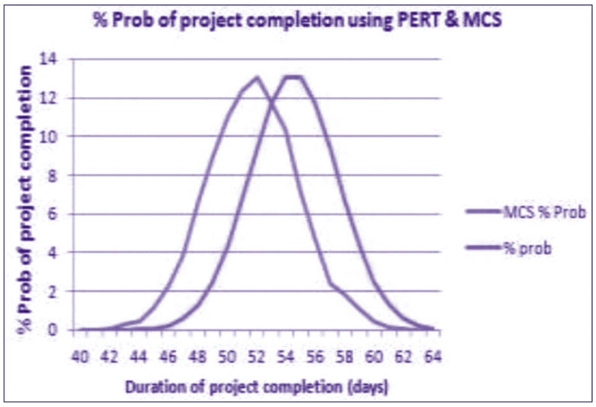
- If P value is less than 0.05 (95% confidence level of the results), then the difference is significant; otherwise, accept the null hypothesis.

- **The % Probability of project completion by both PERT & Monte Carlo simulation is shown in the table 5**

Table 5: % Probability & Cumulative % probability of project completion by PERT & Monte Carlo Simulation

No. of days	MCS %probability	MCS cum %probability	PERT %probability	PERT cum %probability
40	0	0	0.0002	0.0002
41	0.02	0.02	0.0006	0.0007
42	0.10	0.12	0.0024	0.0032
43	0.28	0.4	0.0091	0.0123
44	0.42	0.82	0.0306	0.0429
45	1.12	1.94	0.0921	0.1350

46	2.32	4.26	0.2480	0.3830
47	3.86	8.12	0.5985	0.9815
48	6.7	14.82	1.2935	2.2750
49	8.92	23.74	2.5040	4.7790
50	11.02	34.76	4.3421	9.1211
51	12.36	47.12	6.7444	15.8655
52	13.02	60.14	9.3837	25.2493
53	11.76	71.9	11.6949	36.9441
54	10.34	82.24	13.0559	50.0000
55	7.1	89.34	13.0559	63.0559
56	4.74	94.08	11.6949	74.7507
57	2.36	96.44	9.3837	84.1345
58	1.82	98.26	6.7444	90.8789
59	1.04	99.3	4.3421	95.2210
60	0.42	99.72	2.5040	97.7250
61	0.16	99.88	1.2935	99.0185
62	0.1	99.98	0.5985	99.6170
63	0.02	100	0.2480	99.8650
64	0	100	0.0921	99.9571



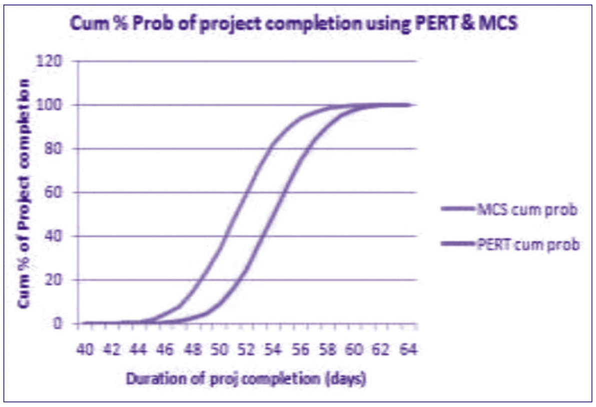


Figure 6: % Probability & Cumulative % Probability of project completion using PERT & Monte Carlo Simulation

Figure 6 shows the % probability & cumulative % probability of project completion using PERT & Monte Carlo simulation.

To conduct the comparison of means, the "Paired" T-test is used. The P-value obtained is 0.0002, which is less than 0.05. **Hence we reject the null hypothesis**. Thus **alternate hypothesis is accepted** i.e. the results obtained from both the methods have a significant difference. The comparison shows that the result obtained from Monte Carlo Simulation is closer to practical project completion duration.

6. CONCLUSION:

For the timely completion of a project, adopting appropriate project scheduling tools & techniques is of utmost importance. This not only helps to plan each activity efficiently but gives the project managers an approximate duration of project completion.

Project scheduling techniques PERT & Monte Carlo simulation is applied on a project and the results obtained by both the techniques are compared in this research study using hypothesis testing (paired t test). It is concluded that there is a significant difference between the results obtained by both the methods. The result obtained using Monte Carlo simulation is closer to the practical duration of project completion. Thus using Monte Carlo simulation, project managers can evaluate the schedule risk analysis of a project by evaluating the criticality index & probability of project completion.

REFERENCES:

1. A. A. Opaleye, O. E. Charles-Owaba & B. Bender (2017) - Relevance of historical data based activity scheduling and Risk Mitigation model. - International Journal of Science and Technology Volume 6 No. 3, March, 2017 ISSN 20497318.
2. A. Connor & S. Mac Donell (2006) - Using historical data in stochastic estimation of software project duration - 19th Annual Conference of the National Advisory Committee on Computing Qualifications (NACCQ 2006), Wellington, New Zealand.
3. Brenda McCabe (2003). - Monte Carlo Simulation for schedule risks. - Simulation Conference, 2003. Proceedings of the 2003 Winter DOI:10.1109/WSC.2003.1261603
4. C RAGSDALE (1989). - The Current State of Network Simulation in Project Management Theory and Practice. OMEGA Int. J. of Mgmt Sci., Vol. 17, No. I, pp. 21-25, 1989-https://doi.org/10.1016/0305-0483(89)90016-9
5. H Arsham (1993) - Managing Project Activity-Duration Uncertainties. OMEGA Int. I. of Mgmt Sci., Vol, 21, No. 1, pp. 111-122 https://doi.org/10.1016/0305-0483(93)90043-K.
6. J. K. Visser (2016) - Suitability of Different Probability Distributions for Performing Schedule Risk Simulations in Project Management. 2016 Proceedings of PICMET '16: Technology Management for Social Innovation. DOI: 10.1109/PICMET.2016.7806608.
7. Karel Doubravský and Radek Dosko?il (2015). - Comparison of Approaches for Calculating the Probability of a Project Completion. - Journal of Eastern Europe Research in Business & Economics Vol. 2015 (2015), Article ID 638688, 7 Pages DOI: 10.5171/2015.638688
8. Konstantinos A. Kirytopoulos, Vrassidas N. Leopoulos and Viktor K. Diamantas(2008). - PERT vs. Monte Carlo Simulation along with the suitable distribution effect. - Int. J. Project Organisation and Management, Vol. 1, No. 1, 2008. DOI: 10.1504/IJPOM.2008.020027.
9. K. R. MacCrimmon & C. A. Ryavec (1963). - An analytical study of the pert Assumptions. - Operations Research 12(1):16-37. February 1964. DOI:10.1287/opre.12.1.16.
10. Liu Jun - Yan (2012). - Schedule Uncertainty Control: A Literature review. - Physics Procedia 33 (2012) 1842 - 1848. https://doi.org/10.1016/j.phpro.2012.05.293.
11. M. A. A. Cox (1995). - Simple normal approximation to the completion time distribution for a PERT network. - International Journal of Project Management Vol. 13, No. 4, pp. 265-270. https://doi.org/10.1016/0263-7863(95)00026-M
12. Mertcan Karabulut (2017). - Application of Monte Carlo simulation and PERT/CPM techniques in planning of construction projects: A Case Study. - Periodicals of Engineering and Natural Scinces ISSN 2303-4521 Vol.5, No.3, December2017, pp. 408?420.
13. M. Hajdu & O. Bokor (2014). - The Effects of Different Activity Distributions on Project Duration in PERT Networks - Procedia - Social and Behavioral Sciences 119 (2014) 766 - 775 DOI: 10.1016/j.sbspro.2014.03.086.
14. M. Hajdu & O. Bokor (2016) - Sensitivity analysis in PERT networks: Does activity duration distribution matter? - Automation in construction 65 (2016) 1-8 http://dx.doi.org/10.1016/j.autcon.2016.01.003.
15. Mira Mileusnic Skrtic and Karolina Horvatincic (2014). - Project Risk Management: Comparative Analysis of Methods for Project Risks Assessment. - Collegium antropologicum, vol 38 supplement 1.
16. Stefania Tattoni & Massimiliano M. Schiraldi (2008). - Massimiliano M. Schiraldi Estimating projects duration in uncertain environments: Monte Carlo simulations strike back. - 22nd IPMA World Congress "Project Management to Run" 9 - 11 November 2008, Roma, Italy.
17. W. Na, P. Wuliang* & G. Hua (2014) - A Robustness Simulation Method of Project Schedule based on the Monte Carlo Method. - The Open Cybernetics & Systemics Journal, 2014, 8, 254-258. DOI: 10.2174/1874110X01408010254.
18. Wolfgang Tysiak (2011). - Risk Management in Projects: The Monte Carlo Approach versus PERT. - The 6th IEEE International Conference on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications 15-17 September 2011, Prague, Czech Republic. DOI: 10.1109/IDAACS.2011.6072904.
19. Wyroz?bski P & Wyroz?bska A. (2013). - Challenges of project planning in the probabilistic approach using PERT, GERT and Monte Carlo. - Journal of Management and Marketing 2013 November 30, Volume 1, Issue 1. ISSN.· 1339-4896
20. Zhaoni Kong, Jianping Zhang, Chao Li, Xiaoying Zheng and Qingqing Guan (2015). - Risk Assessment of Plan Schedule by Monte Carlo Simulation - International Conference on Information Technology and Management Innovation (ICITMI 2015). ISBN-9781510814479

AUTHORS

Pooja Deshmukh, Second Year M. Tech. (Project Management), Department of Production Engineering and Industrial Management, College of Engineering Pune,

Email- poojard16.pm@coep.ac.in

Dr. Mrs. N. R. Rajhans, Professor, Department of Production Engineering and Industrial Management, College of Engineering, Pune,

Email- nrr.prod@coep.ac.in